Chapter 2 Exercise 2.1

Q. 1. (i) $3 \times 2 \times 1 = 6$

(ii) $2 \times 1 + 4 \times 3 \times 2 \times 1 = 26$

(iii) 4,320

(iv) 8

(v) 504

(vi) 10

(vii) 11

(viii) 14

(ix) $\frac{20 \times 19!}{19!} = 20$

(x) $\frac{21 \times 20 \times 19!}{19!} = 420$

Q. 2. $3 \times 7 \times 2 = 42$

Q. 3. $5 \times 2 \times 3 = 30$

Q. 4. $26 \times 10 = 260$ registrations

Q. 5. (a) (i) $8 \neq 5!$ No

(ii) $3 \times 2 \times 1 \times 2 \times 1 \neq 710$ No

(iii) $1 + 1 = 2$ Yes

(b) (i) $3 \times 2 \times 1 = 6$

(ii) $5! = 120$

(iii) $8! = 40{,}320$

Q. 6. (i) 120

(ii) 360

(iii) 720

Q. 7. $4! = 24$

(i) $\boxed{3} \times \boxed{2} \times \boxed{1} \times \boxed{1} = 6$

(ii) $\boxed{3} \times \boxed{2} \times \boxed{1} \times \boxed{3} = 18$

(iii) $\boxed{2} \times \boxed{3} \times \boxed{2} \times \boxed{1} = 12$

Q. 8. (i) ${}^7P_7 = 5{,}040$

(ii) ${}^7P_5 = 2{,}520$

(iii) ${}^7P_2 = 42$

Q. 9. (i) $6! = 720$

(ii) $\boxed{1} \times 5! = 120$

(iii) $\boxed{1} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1} \times \boxed{1} = 24$

(iv) $\boxed{2} \times \boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1} = 240$

(v) $\boxed{4} \times 5! = 480$

Q. 10. (i) $7! = 5{,}040$

(ii) $\boxed{2} \times 6! = 1{,}440$

(iii) $\boxed{5} \times 6! = 3{,}600$

(iv) $6! \times 2 = 1{,}440$

(v) $5{,}040 - 1{,}440 = 3{,}600$

Q. 11. (i) $8! = 40{,}320$

(ii) $1 \times \boxed{6!} \times 1 = 720$

(iii) $6! \times 3! = 720 \times 6 = 4{,}320$

(iv) $7! \times 2 = 10{,}080$

(v) $40{,}320 - 10{,}080 = 30{,}240$

Q. 12. (i) $9! = 362{,}880$

(ii) $7! \times 3! = 30{,}240$

(iii) $4! \times 6! = 17{,}280$

(iv) $8! = 40{,}320$

(v) $\boxed{6} \times 7! \times \boxed{5} = 151{,}200$

Q. 13. (a) (i) $\frac{10 \times 9!}{9!} = 10$

(ii) $\frac{11 \times 10 \times 9 \times 8 \times \cancel{7!}}{\cancel{7!}\ 4!} = 330$

(iii) $\frac{23 \times 22!}{22!} = 23$

(iv) $\frac{13 \times 12 \times 11!}{11!} = 156$

(b) (i) $\frac{n(n-1)!}{(n-1)!} = n$

(ii) $\frac{(n+1)(n!)}{n!} = n + 1$

(iii) $\frac{n!}{(n-1)!} - \frac{n!}{n!}$

$\frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}} - 1$

Ans $= n - 1$

(c) (i) $\boxed{3} \times \boxed{3} \times \boxed{2} \times \boxed{1} = 18$

(ii) $\boxed{3} \times \boxed{4} \times \boxed{4} \times \boxed{4} = 192$

Q. 14. $\boxed{5} \times \boxed{7} \times \boxed{7} = 245$

[since there are 7 numbers left to choose from.]

Q. 15. $\boxed{3} = 3$

$\boxed{3} \times \boxed{2} = 6$

$\boxed{3} \times \boxed{2} \times \boxed{1} = \frac{6}{15}$

Q. 16. $\boxed{3}$ 3 [none can start with 0]

$\boxed{3} \times \boxed{3}$ 9

$\boxed{3} \times \boxed{3} \times \boxed{2}$ 18

$\boxed{3} \times \boxed{3} \times \boxed{2} \times \boxed{1}$ $\frac{18}{48}$

Q. 17. (i) $\boxed{26}\boxed{25} \mid \boxed{10}\boxed{9}\boxed{8} = 468{,}000$

(ii) $\boxed{1}\boxed{25} \mid \boxed{1}\boxed{9}\boxed{8} = 1800$

Q. 18. $\boxed{4} \times \boxed{10} \times \boxed{1} \times \boxed{1} \times \boxed{2} = 80$

Q. 19. 3 Colours $\times \boxed{26} \mid \boxed{10}\boxed{10}\boxed{10} = 78{,}000$

Q. 20. ${}^{40}P_3 = 59{,}280$

Q. 21. $\boxed{2} \times \boxed{4} \times \boxed{4} \times \boxed{1} = 32$

Q. 22. $\boxed{1}\boxed{1}\boxed{1} \mid \boxed{4}\boxed{10}\boxed{5} = 200$

Q. 23. (i) $5! = 120$

(ii) $\boxed{3}\boxed{2}\boxed{2}\boxed{1}\boxed{1} = 12$

Q. 24. $\boxed{G}\boxed{B}\boxed{G}\boxed{B}\boxed{G}\boxed{B}\boxed{G}$

$4 \;\; 3 \;\; 3 \;\; 2 \;\; 2 \;\; 1 \;\; 1 = 144$

Q. 25. (i) $9! = 362{,}880$

(ii) $8! \times 2 = 80{,}640$

(iii) Together $8! \times 2 = 80{,}640$

$362{,}880 - 80{,}640 = 282{,}240$

(iv) $5! \times 4! = 2{,}880$

Q. 26. (i) $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

(ii) $\boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1} \times \boxed{2} = 240$

(iii) $\boxed{4} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1} \times \boxed{1} = 96$

or ②

$\boxed{3} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1} \times \boxed{1} = 72$

④

Ans $= 96 + 72 = 168$

Q. 27. (i) $\boxed{4}\boxed{3}\boxed{2}\boxed{1}\boxed{3} = 72$

(ii) $\boxed{3}\boxed{3}\boxed{2}\boxed{1}\boxed{1} = 18$

⑨

$\boxed{2}\boxed{3}\boxed{2}\boxed{1}\boxed{1} = 12$

⑤

$\boxed{2}\boxed{3}\boxed{2}\boxed{1}\boxed{1} = \frac{12}{42}$

⑦

Ans $= 42$

Exercise 2.2

Q. 1. 45, 165; 35; 14; 190; 84; 84, 78; 3,060; 3,060

Q. 2. $286 + 715 = 1{,}001$

$\therefore 1{,}001 = 1{,}001$ QED

Q. 3. $792 - 330 = 462$

$\therefore 462 = 462$ QED

Q. 4. $(1 + 5 + 10 + 10 + 5 + 11) = 2^k$

$32 = 2^k$

$\boxed{k = 5}$

Q. 5. $2(1716) = 3{,}432$

$\therefore 3{,}432 = 3{,}432$ QED

Q. 6. $55 + 165 + 495 = 715$

$\therefore 715 = 715$ QED

Q. 7. $\binom{15}{11} = 1365$

Q. 8. (i) $\binom{7}{3} = 35$

(ii) $\binom{6}{2} = 15$

(iii) $\binom{6}{3} = 20$

Q. 9. (i) $\binom{13}{4} = 715$

(ii) $\binom{6}{2}\binom{7}{2}$

$= (15)\,(21)$

$= 315$

(iii) $\binom{6}{4}\binom{7}{0} = 15$

Q. 10. (i) $\binom{15}{13} = 105$

(ii) $\binom{14}{12} = 91$

Q. 11. (i) $\binom{8}{5} = 56$

(ii) $\binom{7}{4} = 35$

(iii) $56 - 35 = 21$

(iv) $\binom{6}{3} = 20$

Q. 12. $\binom{4}{4}\binom{7}{1} + \binom{4}{3}\binom{7}{2} = 7 + 84 = 91$

Q. 13. $\binom{13}{2} = 78$

Q. 14. $\binom{12}{2} \times 2 = 132$

Q. 15. (i) $\binom{9}{4} = 126$

(ii) $\binom{5}{5}\binom{4}{0} + \binom{5}{4}\binom{4}{1} + \binom{5}{3}\binom{4}{2}$

$= 1 + 20 + 60 = 81$

(iii) $\binom{5}{1}\binom{4}{4} = 5$

Q. 16. $\binom{45}{6} = 8{,}145{,}060$

Q. 17. (i) $\binom{10}{5} = 252$

(ii) $\frac{252}{2} = 126$

(iii) $\binom{8}{4} = 70$

$\frac{70}{2} = 35$ ways

(iv) $\binom{12}{6} = 924$

$\frac{924}{2} = 462$ ways

Q. 18. $\binom{25}{2} = 300$ or $1 + 2 + 3 + \ldots + 24 = 300$

Q. 19. (i) $\binom{10}{4} = 210$

(ii) $\binom{10}{5} = 252$

(iii) $\binom{11}{5} - \binom{9}{3} = 378$

(all cases MINUS the cases with those two together)

Q. 20. $\binom{13}{4} - \binom{11}{2} = 660$

Q. 21. (i) $\binom{12}{2} = 66$ (ii) $\binom{12}{3} = 220$

Q. 22. (i) $\binom{52}{5} = 2{,}598{,}960$

(ii) $\binom{51}{4} = 249{,}900$

(iii) $2{,}598{,}960 - 249{,}900$

$= 2{,}349{,}060$

(iv) $\binom{48}{1} = 48$

(v) $\binom{48}{2}\binom{4}{3} = 4{,}512$

Q. 23. $\binom{n}{2} = 210$

$\frac{n(n-1)}{2} = 210$

$n^2 - n = 420$

$(n - 21)(n + 20)$

$\boxed{n = 21}$ $\quad n = -20$

Q. 24. $\frac{n(n-1)(n-2)}{6} = 15n$

$n^2 - 3n + 2 = 90$

$n^2 - 3n - 88 = 0$

$(n + 8)(n - 11)$

$\boxed{n = 11}$

$\binom{11}{3} = 15(11)$

$165 = 165$

Q. 25. $3\left(\frac{x(x-1)(x-2)}{3 \times 2 \times 1}\right)$

$= 2\left(\frac{x(x-1)}{2 \times 1}\right) + 5(x)$

$\frac{x^2 - 3x + 2}{2} = x - 1 + 5$

$x^2 - 3x + 2 = 2x + 8$

$x^2 - 5x - 6 = 0$

$(x + 1)(x - 6) = 0$

$\boxed{x = 11}$

$3\binom{6}{3} = 2\binom{6}{2} + 5\binom{6}{1}$

$3(20) = 30 + 30$

$60 = 60$

Exercise 2.3

Q. 1.

Probability	Event
Fifty-fifty	C
Certain	E
Very unlikely	B
Impossible	A
Very likely	D

Q. 2. (i) $\frac{40}{50} = 0.8$

(ii) $60 \times \frac{40}{50} = 48$

Q. 3.

Event	Letter
Getting tails on single flip of a fair coin	R
Getting a 4 on a single roll of a fair six sided die	Q
Getting a 7 on a single roll of a fair six sided die	P
A person being born on a day that ends with the letter y	T
A person not being born on a Sunday	S

Q. 4. (i) $\frac{22}{167} \times \frac{100}{1} = 0.1317$ (ii) $\frac{3}{20} = 0.15$

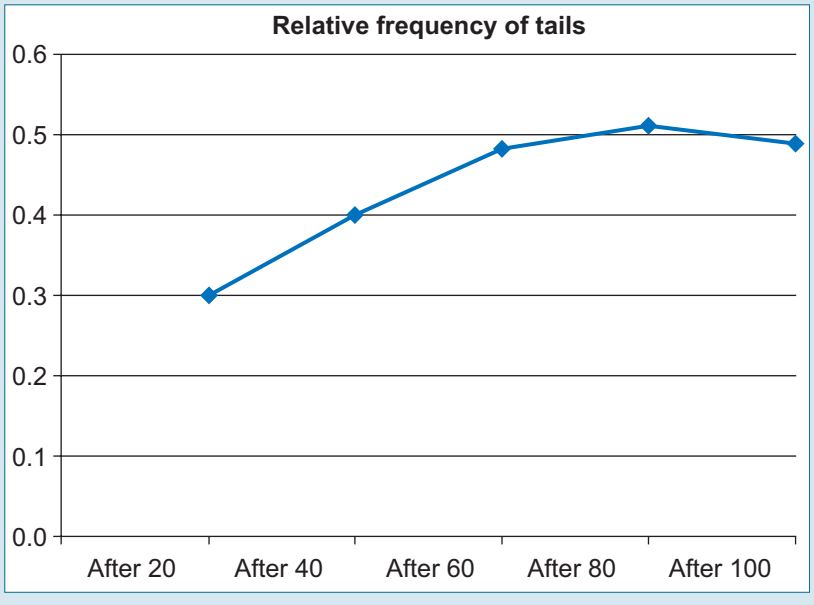
Ans = 13%

Conor's punctuality has disimproved. He is now late approximately 15% of the time compared to 13% of the time last year.

Q. 5. (i)

Number of flips	After 20	After 40	After 60	After 80	After 100
Number of heads	14	24	31	39	51
Number of tails	6	16	29	41	49
Relative frequency of tails	0.3	0.4 $\left[\frac{16}{40}\right]$	0.4833 $\left[\frac{29}{60}\right]$	0.5125 $\left[\frac{41}{80}\right]$	0.49 $\left[\frac{49}{100}\right]$

(ii)



(iii) Yes, Beth is correct because the more an experiment is repeated, the closer the relative frequency is to the actual probability.

Q. 6. (i) $\frac{161}{211} = 0.7630$

Ans = 76%

(ii) $46 \times \frac{161}{211} = 35.0995$

Ans = 35

(iii) Pundit A is most accurate
Pundit B is least accurate

Q. 7. (i) $\frac{210}{365} = \frac{42}{73} = 0.5753$

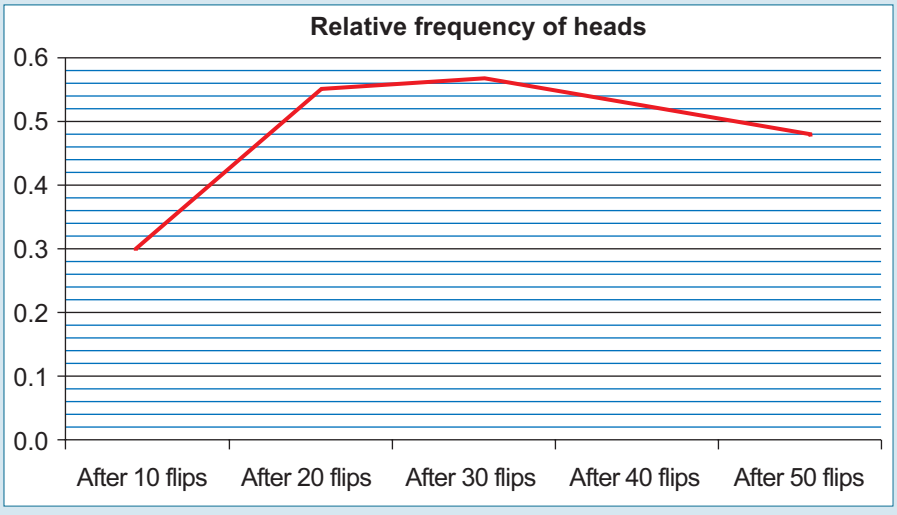
Ans = 57.5%

(ii) Mr and Mrs Abercrombie would have expected $21 \times 0.573 \approx 12$ days of sunshine out of 21, based on the relative frequency of sunny days. They did get this number of sunny days so their complaint is not reasonable.

Q. 8. (i)

	After 10 flips	After 20 flips	After 30 flips	After 40 flips	After 50 flips
Relative frequency of heads	0.3	0.55	0.5666	0.525	0.48

(ii)



(iii) No, as one would expect that the relative frequency gets closer and closer to the actual probability of the event as the experiment is repeated, which is what happens in this case. After 50 flips, the relative frequency of a head turning up is 0.48, which is very close to the actually probability of 0.5.

Q. 9. (i) $\frac{94}{179} = 0.5251$

Ans = 52.5%

(ii) $\frac{57 \times 94}{179} = 30.88$

(iii) It satisfies me, as it is in line with the relative frequency.

(iv) No.

Ans = 31 successes

Q. 10. (i) Cathal's data will give the best estimate, as the more an experiment is repeated, the closer the relative frequency is to the actual probability.

(ii) $160 \times \left(\frac{1}{8}\right) = 20$ times

(iii) The total result of the students is different, as this is the experimental probability. It is an estimate of the true probability. While it may not be exactly the same as the theoretical probability, with repeated trialling it will get very close to it.

Q. 11. (i)

First digit on data	Frequency	Relative frequency (out of 1500) as a decimal
1	444	0.296
2	255	0.170
3	165	0.110
4	141	0.094
5	117	0.078
6	108	0.072
7	98	0.065
8	85	0.057
9	87	0.058

(ii) Yes

(iii) Because before you eat 7 sandwiches, you must first eat 1, 2, 3, 4 and 6. Before you reach 90 years of age, you must first be 1, 2, 3 etc. Hence, low numbers are more common than higher numbers.

Exercise 2.4

Q. 1. $\frac{1}{2}$

Q. 2. (i) $\frac{3}{6} = \frac{1}{2}$

(ii) $\frac{4}{6} = \frac{2}{3}$

(iii) $\frac{3}{6} = \frac{1}{2}$

Q. 3. 3 Vowels

(i) $\frac{3}{8}$

(ii) $\frac{3}{8}$

(iii) $\frac{5}{8}$

Q. 4. 14 Boys 16 Girls Total 30

3 Boys/Glasses 2 Girls/Glasses

(i) $\frac{16}{30} = \frac{8}{15}$

(ii) $\frac{14}{30} = \frac{7}{15}$

(iii) $\frac{5}{30} = \frac{1}{6}$

(iv) $\frac{3}{30} = \frac{1}{10}$

Probability of girls who wears glasses

$= \frac{2}{30} = \frac{1}{15}$

$180 \times \frac{1}{15} = 12$

Ans = 12 days

Q. 5. (i) $\frac{13}{52} = \frac{1}{4}$

(ii) $\frac{26}{52} = \frac{1}{2}$

(iii) $\frac{4}{52} = \frac{1}{13}$

(iv) $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

(v) P(diamond or King) = P(D) + P(K) – P(king of Diamonds)

$= \frac{1}{4} + \frac{1}{13} - \frac{1}{52} = \frac{4}{13}$

(vi) $P(B \cup K) = P(B) + P(K) - P(B \cap K)$

$= \frac{1}{2} + \frac{1}{13} - \frac{2}{52} = \frac{7}{13}$

Q. 6. (i) $\frac{2}{7}$ (iii) $\frac{7}{7} = 1$

(ii) $\frac{4}{7}$ (iv) 0

Q. 7. (i) Yes (vi) No

(ii) No (vii) No

(iii) Yes (viii) Yes

(iv) No (ix) No

(v) Yes (x) Yes

Q. 8. P(E) = 0.4

P(F) = 0.6

$P(E \cap F) = 0.1$

$P(E \cup F) = 0.9$

$P(E \cup F) = P(A) + P(B) - P(A \cap B)$

0.4 + 0.6 – 0.1 = 0.9 QED

Q. 9. (a) From the Venn diagram $\#\cup = 20$

(i) $P(A) = \frac{10}{20} = \frac{1}{2}$

(ii) $P(B) = \frac{5}{20} = \frac{1}{4}$

(iii) $P(A \cap B) = \frac{3}{20}$

(iv) $P(A \cup B) = \frac{1}{2} + \frac{1}{4} - \frac{3}{20} = \frac{3}{5}$

(v) $P(A \setminus B) = \frac{7}{20}$

(vi) $\frac{7}{20} + \frac{2}{20} = \frac{9}{20}$

(vii) $P(A') = \frac{1}{2}$

(viii) $P(A \cup B)' = \frac{2}{5}$

(b) (i) $\frac{7}{20} = \frac{10}{20} - \frac{3}{20}$

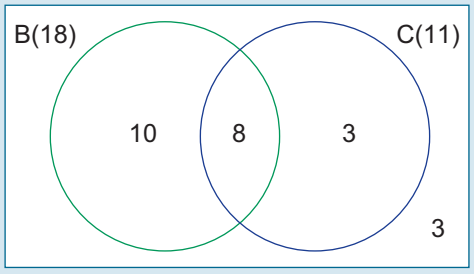
$\frac{7}{20} = \frac{7}{20}$ QED

(ii) $\frac{3}{5} = \frac{10}{20} + \frac{5}{20} - \frac{3}{20}$

$\frac{3}{5} = \frac{12}{20}$

$\frac{3}{5} = \frac{3}{5}$ QED

Q. 10.



$\#(B \cap C) = (18 + 11 + 3) - 24 = 8$

(i) $P(B) = \frac{18}{24} = \frac{3}{4}$

(ii) $P(C) = \frac{11}{24}$

(iii) $P(B \cap C) = \frac{8}{24} = \frac{1}{3}$

(iv) $P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$= \frac{3}{4} + \frac{11}{24} - \frac{1}{3}$

$= \frac{7}{8}$

(v) $P(B') = \frac{6}{24} = \frac{1}{4}$

(vi) F (ix) T

(vii) T (x) T

(viii) F

Q. 11. (a) (i) $P(Q) = \frac{1}{13}$

(ii) $P(R) = \frac{1}{2}$

(iii) $P(S) = \frac{1}{4}$

(iv) $P(Q \cap R) = \frac{2}{52} = \frac{1}{26}$

(v) $P(R \setminus Q) = \frac{24}{52} = \frac{6}{13}$

(vi) $P(R \cup Q) = P(R) + P(Q) - P(R \cap Q)$

$= \frac{1}{2} + \frac{1}{13} - \frac{1}{26}$

$= \frac{14}{26}$

$= \frac{7}{13}$

(vii) $P(S') = \frac{39}{52} = \frac{3}{4}$

(viii) $P(Q \cup R \cup S)$ Red Spade Red Green Green Spade

$= \frac{4}{52} + \frac{26}{52} + \frac{13}{52} - \frac{0}{52} - \frac{2}{52} - \frac{1}{52}$

$= \frac{40}{52} = \frac{10}{13}$

(b) (i) $\frac{7}{23} = \frac{1}{2} + \frac{1}{13} - \frac{2}{52}$

$\frac{7}{13} = \frac{7}{13}$ QED

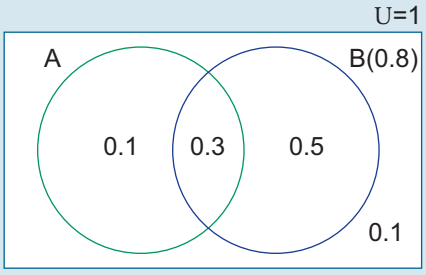
(ii) $\frac{6}{13} = \frac{1}{2} - \frac{1}{26}$

$\frac{6}{13} = \frac{6}{13}$ QED

(iii) $\frac{3}{4} = 1 - \frac{13}{52}$

$\frac{3}{4} = \frac{3}{4}$ QED

Q. 12.



(a) (i) $P(A) = 0.4$

(ii) $P(A \cup B) = 0.9$

(b) $0.9 = 0.4 + 0.8 - 0.3$

$0.9 = 0.9$

Q. 13.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

(i) $\frac{3}{36} = \frac{1}{12}$

(ii) $\frac{6}{36} = \frac{1}{6}$

(iii) $\frac{6}{36} = \frac{1}{6}$

(iv) $\frac{18}{36} = \frac{1}{2}$

(v) $\frac{18}{36} = \frac{1}{2}$

(vi) 4, as this is twice as great as 2

6, as this is twice as great as 3

8, as this is twice as great as 4

10, as this is twice as great as 5

12, as this is twice as great as 6

Ans $= \frac{17}{36}$

(vii) $\frac{33}{36} = \frac{11}{12}$

Q. 14. S = {(H,H,H),(H,H,T),(H,T,T),(H,T,H), (T,H,H),(T,H,T),(T,T,H), (T,T,T)}

(i) 8

(ii) $\frac{1}{8}$

(iii) $\frac{7}{8}$

(iv) $\frac{1}{2}$

Q. 15.

	1	2	3	4	5	6
H	H 1	H 2	H 3	H 4	H 5	H 6
T	T 1	T 2	T 3	T 4	T 5	T 6

(i) $\frac{1}{12}$

(ii) $\frac{3}{12} = \frac{1}{4}$

(iii) $\frac{1}{2}$

Q. 16.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

(a) (i) $\frac{4}{20} = \frac{1}{5}$

(ii) $\frac{14}{20} = \frac{7}{10}$

(iii) $\frac{10}{20} + \frac{10}{20} - \frac{5}{20} = \frac{15}{20} = \frac{3}{4}$

(iv) $\frac{4}{20} + \frac{10}{20} - \frac{2}{20} = \frac{12}{20} = \frac{3}{5}$

(v) $\frac{5}{20} = \frac{1}{4}$

(b) Mutually exclusive if $P(\text{multiple of } 7 \cap G) = \varnothing$

So, yes, the events are mutually exclusive (no green tickets are multiples of 7).

Q. 17. $\frac{n}{(11 + 13 + n)} = \frac{1}{4}$

$\therefore 4n = 24 + n$

$\therefore 3n = 24$

$\therefore n = 8$

Q. 18. (i) $\frac{33}{50}$

(ii) $\frac{20}{50} = \frac{2}{5}$

(iii) $\frac{5}{50} = \frac{1}{10}$

(iv) $\frac{12}{50} + \frac{15}{50} = \frac{27}{50}$

Q. 19. (i) 216 (ii) $\frac{1}{216}$

(iii) 662, 653, 644, 626, 635

Ans $= \frac{5}{216}$

(iv) 662, 626, 266, 653, 635, 644, 464, 446, 455

Ans $= \frac{9}{216} = \frac{1}{24}$

Q. 20. (i) $\frac{31}{365} = 0.0849 \approx 0.085$

(ii) $\frac{28}{365} = 0.0767 \approx 0.077$

(iii) $\frac{(31 + 28)}{365} = 0.1616 \approx 0.162$

Q. 21. (i) $x = 1 - 0.3 - 0.2 - 0.4 = 0.1$

(ii) No

(iii) $P(E) = 0.4$

$P(F) = 0.3$

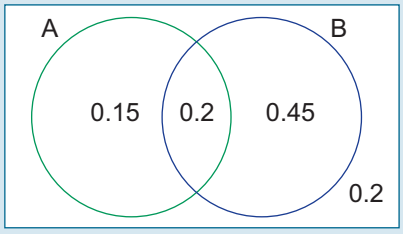
$P(E \cap F) = 0.1$

$P(E \cup F) = 0.4 + 0.3 - 0.1 = 0.6$

(iv) $0.6 = 0.4 + 0.3 - 0.1$

$0.6 = 0.6$ QED

Q. 22.



(i) 0.65

(ii) 0.35

(iii) $0.8 = 0.35 + 0.65 - 0.2$

$0.8 = 0.8$ QED

Q. 23.

	(0,1)	(0,2)	(0,3)	(0,4)
(1,0)	$\sqrt{2}$	$\sqrt{5}$	$\sqrt{10}$	$\sqrt{17}$
(2,0)	$\sqrt{5}$	$\sqrt{8}$	$\sqrt{13}$	$\sqrt{20}$
(3,0)	$\sqrt{10}$	$\sqrt{13}$	$\sqrt{18}$	5
(4,0)	$\sqrt{17}$	$\sqrt{20}$	5	$\sqrt{32}$
(5,0)	$\sqrt{26}$	$\sqrt{29}$	$\sqrt{34}$	$\sqrt{41}$

(i) $\frac{2}{20} \times \frac{100}{1} = 10\%$

(ii) $\frac{5}{20} \times 100 = 25\%$

(iii) $\frac{4}{20} \times 100 = 20\%$

Q. 24. (i) 0.13

(ii) 0.6

(iii) 0.36

(iv) 0.3

$P(B \cup C) = P(B) + P(C) - P(B \cap C)$

$0.7 = 0.39 + 0.41 - 0.1$

$0.7 = 0.7$ QED

Q. 25. (a) (i) $k = 0.08$

(ii) $P(X \cup Z) = 1 - 0.05 - 0.3$

$= 0.65$

(iii) $P(X \cap Z) = 0.07 + 0.08$

$= 0.15$

(iv) $P(Y \setminus (X \cup Z)) = 0.3$

(v) $P(Z') = 0.2 + 0.1 + 0.3 + 0.05$

$= 0.65$

(b) (i) $0.75 = 0.53 + 0.35 - 0.13$

$0.75 = 0.75$ QED

(ii) $0.47 = 1 - 0.53$

$0.47 = 0.47$ QED

Q. 26. (a)

	6		7		8		9		10	
1	1	6	1	7	1	8	1	9	1	10
2	2	6	2	7	2	8	2	9	2	10
3	3	6	3	7	3	8	3	9	3	10
4	4	6	4	7	4	8	4	9	4	10
5	5	6	5	7	5	8	5	9	5	10

(b) ***SUM***

	6	7	8	9	10
1	7	8	9	10	11
2	8	9	10	11	12
3	9	10	11	12	13
4	10	11	12	13	14
5	11	12	13	14	15

PRODUCT

	6	7	8	9	10
1	6	7	8	9	10
2	12	14	16	18	20
3	18	21	24	27	30
4	24	28	32	36	40
5	30	35	40	45	50

(i) $P(E) = \frac{3}{25}$

$P(F) = \frac{14}{25}$

(ii) Yes, because the probability of getting two numbers as described in this question whose sum is less than 9 but whose product is greater than 20 is zero.

(iii) $P(E \cup F) = \frac{3}{25} + \frac{14}{25} = \frac{17}{25}$

(iv) No, it is not a fair game, as Kevin has a $\frac{13}{25}$ chance of winning.

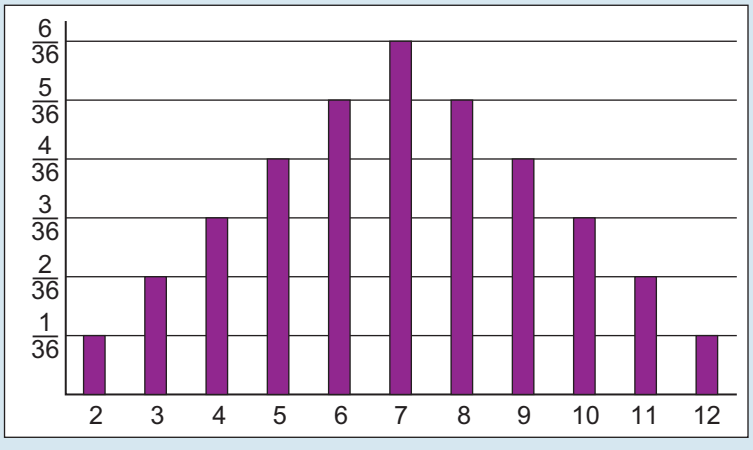
Q. 27. ***SUM***

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

(i)

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

(ii)



(iii) $P(4,5,6 \text{ or } 7) = \frac{(3 + 4 + 5 + 6)}{36} = \frac{18}{36}$

Yes, this is a fair game.

(iv) {7,8,9,10}

{4,5,6,7}

(v)

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$
Angle	10	20	30	40	50	60	50	40	30	20	10

Exercise 2.5

Q. 1. (i) $P(E|F) = \frac{P(E \cap F)}{P(F)}$

$= \frac{0.1}{0.4}$

$= 0.25$

(ii) $P(F|E) = \frac{P(E \cap F)}{P(E)}$

$= \frac{0.1}{0.6} = \frac{1}{6}$

$= 0.1667$

Q. 2.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

(a) $A = \{(6,3),(6,4),(6,5),(6,6),(5,4),(5,5),(5,6),(4,5),(4,6),(3,6)\}$

$B = \{(6,1),(6,2),(6,3),(6,4),(6,5),(6,6),(1,6),(2,6),(3,6),(4,6),(5,6)\}$

$A \cap B = \{(6,3),(6,4),(6,5),(6,6),(3,6),(4,6),(5,6)\}$

(b) (i) $P(B|A) = \frac{7}{10}$

(ii) $P(A|B) = \frac{7}{11}$

Q. 3. (i) $P(H \cap W) = 0.6 \times 0.5 = 0.3$

$P(A \cap W) = 0.2 \times 0.5 = 0.1$

$P(W) = 0.1 + 0.3 = 0.4$

$P(H|W) = \frac{P(H \cap W)}{P(W)}$

$= \frac{0.3}{0.4}$

$= 0.75$

(ii) $P(A|W) = \frac{P(A \cap W)}{P(W)}$

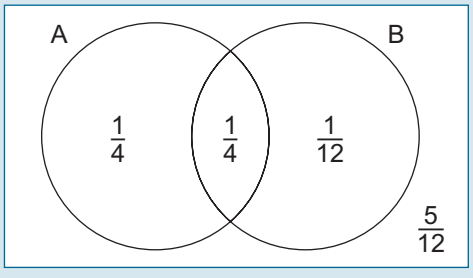
$= \frac{0.1}{0.4}$

$= 0.25$

Q. 4. {GGG,GGB,GBB,BBB,BBG,BGG,BGB, GBG}

$P(G|\text{at least two girls}) = \frac{0.125}{0.5} = 0.25$

Q. 5.



(i) $P(A|B) = \frac{0.25}{0.3333}$

$= 0.75$

(ii) $P(B|A) = \frac{0.25}{0.5}$

$= 0.5$

(iii) $P(A'|B) = \frac{0.0833}{0.3333}$

$= 0.25$

(iv) $P(A|B') = \frac{0.25}{0.6666}$

$= 0.375$

(v) $P(A \cup B) = 0.25 + 0.25 + 0.0833$

$= \frac{7}{12}$

$= 0.5833$

(vi) $P[A|(A \cup B)] = \dfrac{0.5}{0.5833}$

$= \dfrac{6}{7}$

$= 0.8571$

Q. 6.

	Hockey	No Hockey	
Boys	50	50	100
Girls	120	30	150
	170	80	250

	Hockey	No Hockey	
Boys	0.2	0.2	0.4
Girls	0.48	0.12	0.6
	0.68	0.32	1

(i) $P(G|H) = \dfrac{0.48}{0.68}$

$= 0.7059$

(ii) $P(H|G) = \dfrac{0.48}{0.6}$

$= 0.8$

Q. 7.

	Local	No Radio	
Town	0.35	0.35	0.7
Rural	0.06	0.24	0.3
	0.41	0.59	1

$P(T|R) = \dfrac{0.35}{0.41}$

$= 0.8537$

Q. 8. $P(A) = 0.5 \quad P(A \cup B) = 0.6 \quad P(A|B) = 0.75$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Let $P(A \cap B) = y \quad P(B) = x$

$0.75 = \dfrac{y}{x}$.....①

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.6 = 0.5 + x - y$.....②

①..... $y = 0.75x$

②..... $0.6 = 0.5 + x - 0.75x$

$0.1 = 0.25x$

$x = \dfrac{0.1}{0.25}$

$x = 0.4$

$P(B) = 0.4$

Q. 9. (i) $P(E|F) = P(E \cap F)/P(F)$

$\therefore P(E \cap F) = P(E|F).P(F)$

$= \left(\dfrac{1}{9}\right)\left(\dfrac{1}{2}\right)$

$= \dfrac{1}{18}$

(ii) $P(F|E) = \dfrac{\frac{1}{18}}{\frac{2}{5}}$

$= \dfrac{5}{36}$

(iii) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= \dfrac{2}{5} + \dfrac{1}{2} - \dfrac{1}{18}$

$= \dfrac{38}{45}$

Q. 10. Full Sample Space $= \{(B,B), (B,G), (G,B), (G,G)\}$

One is a girl $\Rightarrow$ Sample Space $= \{(B,G), (G,B), (G,G)\}$

(i) P(other a boy) $= \dfrac{2}{3}$

(ii) P(other a girl) $= \dfrac{1}{3}$

Q. 11.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(E) = \dfrac{6}{36} = \dfrac{1}{6}$

$P(F) = \dfrac{11}{36}$

$P(E \cap F) = \dfrac{2}{36}$

$P(F|E) = \dfrac{\frac{2}{36}}{\frac{6}{36}} = \dfrac{1}{3}$

$P(E|F) = \dfrac{\frac{2}{36}}{\frac{11}{36}} = \dfrac{2}{11}$

Q. 12. $P(A) = \dfrac{5}{11}$

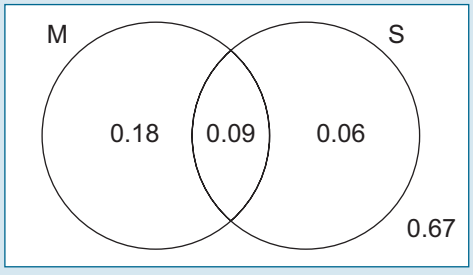
$P(B) = \dfrac{5}{11}$

$$P(A \cap B) = \frac{2}{11}$$

$$P(A|B) = \frac{\frac{2}{11}}{\frac{5}{11}} = \frac{2}{5}$$

$$P(B|A) = \frac{\frac{2}{11}}{\frac{5}{11}} = \frac{2}{5}$$

Q. 13.



(i) $P(M|S) = \frac{0.09}{0.15}$

$= 0.6$

(ii) $P(S|M) = \frac{0.09}{0.27}$

$= \frac{1}{3}$

(iii) $P(S \cup M) = 0.33$

(iv) $P(S \cup M)' = 0.67$

Q. 14. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

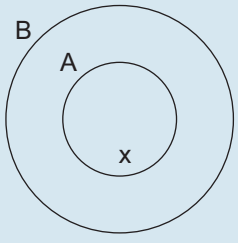
$\therefore \frac{3}{5} = \frac{3}{10} + \frac{1}{2} - P(A \cap B)$

$\therefore P(A \cap B) = \frac{1}{5} = 0.2$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{5}}{\frac{1}{2}} = \frac{2}{5} = 0.4$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{5}}{\frac{3}{10}} = \frac{2}{3}$$

Q. 15.



$P(A \cap B) = x$

$P(A) = x$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$= \frac{x}{x}$

$= 1$

Q. 16. (i) 0.66

(ii) 0.16

(iii) $P(B) = 0.09 + 0.07 + 0.04 + 0.1 = 0.3$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$= \frac{0.16}{0.3}$

$= \frac{8}{15}$

(iv) $P(B|A) = \frac{0.16}{0.66}$

$= \frac{8}{33}$

(v) $P(A|(B \cup C)) = \frac{P(A \cap B \cup C)}{P(B \cup C)}$

$= \frac{0.26}{0.55}$

$= \frac{26}{55}$

(vi) $P(C|(A \cap B)) = \frac{0.07}{0.16}$

$= 0.4375$

Q. 17. (i) $P(A) = 0.3$

$0.3 = 0.09 + 0.07 + 0.04 + x$

$x = 0.1$

$y = 1 - (0.09 + 0.07 + 0.1 + 0.04 + 0.16 + 0.3 + 0.04)$

$= 0.2$

(ii) $P(B|C) = \frac{P(B \cap C)}{P(C)}$

$= \frac{0.26}{0.6}$

$= \frac{13}{30}$

$P(C|B) = \frac{0.26}{0.37}$

$= \frac{26}{37}$

(iii) $P(B|(A \cup C)) = \frac{0.33}{0.76}$

$= \frac{33}{77}$

(iv) $P((A \cap C)|(A \cup B \cup C))$

$= \frac{P((A \cap C) \cap (A \cup B \cup C))}{P(A \cup B \cup C)}$

$= \frac{0.14}{0.8}$

$= 0.175$

(v) $P((A \setminus B) \mid (A \cup B)) = \frac{0.13}{0.5}$

$= 0.26$

Q. 18. $P(B \mid A) = \frac{P(B \cap A)}{P(A)}$

$\therefore 0.5 = \frac{0.2}{P(A)}$

$P(A) = \frac{0.2}{0.5}$

$P(A) = 0.4$

(i) $x = 0.4 - 0.2 = 0.2$

$y = 1 - 0.2 - 0.2 - 0.3 = 0.3$

(ii) $P(A \mid B) = \frac{0.2}{0.5} = 0.4$

Q. 19. (i) $x = 0.08$

(ii) $P(A \mid B) = \frac{0.19}{0.49} = \frac{19}{49}$

(iii) $P(A \mid (B \cup C)) = \frac{0.26}{0.86}$

$= \frac{13}{43}$

(iv) $P((B \cap C) \mid A) = \frac{0.08}{0.3}$

$= \frac{4}{15}$

Q. 20. (i) 5 possibilities of $\geqslant$ 3 boys

BBBB, BBBG, BBGB, BGBB, GBBB

$\therefore P(B \mid 3 \text{ Boys}) = \frac{1}{5}$

(ii) False, the fact that you have had three boys in no way affects the probability of having a girl

Revision Exercises

Q. 1. (a) (i) $7! = 5040$

(ii) $1 \times 5 \times 4 \times 3 \times 2 \times 1 \times 1 = 240$

(b)

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

(i) $\frac{1}{36}$

(ii) $\frac{11}{36}$

(iii) $\frac{5}{36}$

Q. 2. (i) ${}^8P_3 = 336$

(ii) ${}^8C_3 = 56$

(iii) $\frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} = \frac{6}{336} = \frac{1}{56}$

(iv) $1 - \frac{1}{56} = \frac{55}{56}$

Q. 3. (i) $k = 1 - 0.1 - 0.3 - 0.4 = 0.2$

(ii) $P(X) = 0.4$

$P(Y) = 0.7$

$P(X \cap Y) = 0.3$

(iii) $0.8 = 0.4 + 0.7 - 0.3$

$0.8 = 0.8$ QED

(iv) $P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)}$

$= \frac{0.3}{0.7}$

$= \frac{3}{7}$

(v) $P(Y \mid X) = \frac{0.3}{0.4}$

$= \frac{3}{4}$

No, they are not equal, as the probabilities above are different.

Q. 4. (i) P(Sum 3) = 1 and 2 or 2 and 1

$= \left(\frac{1}{4}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{3}\right)$

$= \frac{2}{12} + \frac{1}{12}$

$= \frac{3}{12}$

$= \frac{1}{4}$

P(product 3) = 3 and 1

$= \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)$

$= \frac{1}{6}$

You are more likely to get a sum of 3.

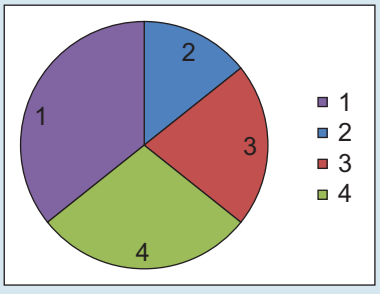
(ii) P(sum of 2) $= \left(\frac{1}{4}\right)\left(\frac{1}{3}\right) = \left(\frac{1}{12}\right)$

P(sum of 3) $= \frac{1}{4}$ (as above)

P(sum of 4) $= \left(\frac{1}{4}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \frac{1}{3}$

P(sum of 5) $= \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) = \frac{2}{6} = \frac{1}{3}$

(iii)



Q. 5. (a) $3 \times 4 \times 3 \times 2 = 72$

(b) (i) $7! = 5{,}040$

(ii) $1 \times 5 \times 4 \times 3 \times 2 \times 1 \times 3 = 360$

(iii) $5 \times 4 \times 3 \times 2 \times 1 \times (3 \times 2 \times 1) = 720$

(iv) $\frac{720}{5{,}040} = \frac{1}{8}$

Q. 6. (i) ${}^{10}C_3 = 120$

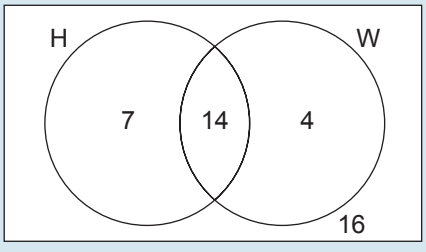
(ii) $\binom{10}{3} - \binom{8}{1} = 112$

(iii) $\frac{120 - 112}{120} = \frac{8}{120} = \frac{1}{15}$

Q. 7. (i) $\frac{14}{21} = \frac{2}{3}$

(ii) $\frac{4}{20} = \frac{1}{5}$

(iii)



$P(H \cap W) = \frac{14}{18} = \frac{7}{9}$

Q. 8. (i) $6! = 720$

(ii) $4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1 = 144$

(iii) $\frac{144}{720} = \frac{1}{5}$

Q. 9. (i) ${}^7C_4 = 35$

(ii) ${}^6C_3 = 20$ [*f* has already been chosen so there are only 3 letters left to pick from the remaining 6]

(iii) $35 - 20 = 15$

(iv) $\frac{20}{35} = \frac{4}{7}$

Q. 10. (i) Relative frequency of a mugging is $\frac{25}{365} = \frac{5}{73} = 0.0684$

January relative frequency is $\frac{2}{31} = 0.0645$

Things have improved slightly: the relative frequency of a mugging in January has decreased by 0.0039, which is 0.39%.

(ii) Relative frequency $\frac{25}{365} \times 0.80$

$= \frac{4}{73} = 0.0548$

$365 \times 0.0548 = 20$ muggings in a year

$\therefore$ 10 in 6 months.

Q. 11. (a) (i) $P(G) = \frac{50}{80} = \frac{5}{8} = 0.625$

(ii) $P(\text{Not Green}) = \frac{30}{80} = 0.375$

(b) $\frac{n(n-1)}{2} = 91$

$\therefore n^2 - n = 182$

$\therefore n^2 - n - 182 = 0$

$\therefore (n - 14)(n + 13) = 0$

$\therefore n = 14$ (Reject $n = -13$)

Q. 12. (i) {HH1,HH2,HH3,HH4,
HT1,HT2,HT3,HT4,
TH1,TH2,TH3,TH4,
TT1,TT2,TT3,TT4,}

(ii) $\frac{1}{16}$

(iii) $\frac{2}{16} = \frac{1}{8}$

(iv) $\frac{4}{16} = \frac{1}{4}$

Q. 13. (i) ${}^{9}C_{3} = 84$

(ii) $\binom{5}{2}\binom{4}{1} = 6 \times 5 = 30$

(iii) $\frac{1}{84}$

(iv) She is looking for the letters of her name but the order is not specified.

So the probability of getting the letters S,U or E in any order is:

$\frac{1}{84} = 0.0119$

Her statement is false: she has a little over a 1% chance of getting the letters of her name if she chooses letters at random from the word.

Q. 14. (i) FHM is the only name that is acceptable.

(ii) ${}^{26}C_{3} = 2{,}600$

Since when you CHOOSE three letters, there is only one alphabetic order in which to arrange them.